

AD-A071 659

ATOMIC ENERGY RESEARCH ESTABLISHMENT HARWELL (ENGLAND)

F/G 18/8

THE CALCULATION OF NEUTRON YIELDS IN MIXTURES AND COMPOUNDS FRO--ETC(U)

FEB 79 D WEST

AERE-R-9372

NL

UNCLASSIFIED

OF
AD
A071659



END
DATE
FILMED
8-79
DDC

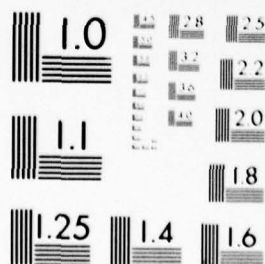
1 OF 1

ADA 071659

D

A071659

FILE COPY



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

HL79/461

6 THE CALCULATION OF NEUTRON YIELDS IN MIXTURES AND COMPOUNDS FROM THE THICK TARGET (α, n) YIELDS IN THE SEPARATE CONSTITUENTS,

10 D. West

ABSTRACT

A simple relation is derived between the thick target yield of neutrons produced following (α, n) reactions in compounds or homogeneous mixtures and the yields in the separate constituents.

alpha It is necessary to assume that the Bragg law of additivity of stopping powers holds for the compound and to make the approximation that the stopping powers of all materials vary with energy in the same way apart from constant factors.

The range of validity of the second hypothesis is examined in some detail. It is shown to be far superior to the approximation of a stopping power independent of energy.

Nuclear Physics Division,
A.E.R.E., HARWELL.

Feb 1979

HL 79/461 (C17)
IMD.

Accession For	
NTIS GRA&I	<input checked="" type="checkbox"/>
DDC TAB	<input checked="" type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	<input type="checkbox"/>
By	
Distribution/	
Availability Codes	
Dist	Avail and/or special
A	

12 19p

046 500

CONTENTS

	<u>Page</u>
Introduction	3
Method	3
References	7

ILLUSTRATIONS

FIGURE

1 Contours of equal departure from $P=1$ where

$$P = \frac{\frac{dE/d(\rho x)_{1,E}}{dE/d(\rho x)_{j,5\text{MeV}}} / \frac{dE/d(\rho x)_{\text{oxygen},E}}{dE/d(\rho x)_{\text{oxygen},5\text{MeV}}}$$

The surface $P=1$ corresponds to a constant value of $K_{j,\text{oxygen}}$ where, as defined in the text,

$$(dE/dx)_j = K_{j1} (dE/dx)_1.$$

Introduction

The production of neutrons from actinide α -particle emitters in contact with light elements is a problem often encountered in present nuclear technology. Usually the medium is a complex mixture or compound and even if the cross-sections for neutron production are known as a function of energy, or the thick target yields are known for all the constituents, it is not a straightforward matter to evaluate the thick target yield in the compound or mixture. It is the purpose of this article to point out one practical method of doing so.

Method

Consider first the separate constituents having, in isolation, n_j atoms/c.c. and the mixture or compound having n'_j atoms per c.c. of each of the j constituents. The total yields of neutrons per α -particle for the separate materials are

$$Y_j = n_j \int_0^{R_j} \sigma_j(E) dx \quad (1)$$

where R_j are the ranges of the α -particles and $\sigma_j(E)$ are the cross sections for the (α, n) reactions at energy E . This can be written

$$Y_j = n_j \int_{E_\alpha}^0 \sigma_j(E) (dx/dE)_j dE \quad (2)$$

where E_α is the initial energy of the α -particle and $(dx/dE)_j$ is the reciprocal of the stopping power, more usually found in tables as $dE/d(\rho_j x)$, where ρ_j is the density.

The yield for a mixture or compound can be written

$$Y_{\text{mixture}} = \sum_j n'_j \int_{E_\alpha}^0 \sigma_j(E) (dx/dE)_{\text{mixture}} dE \quad (3)$$

The problems in using this expression arise because both $\sigma_j(E)$ and $(dx/dE)_{\text{mixture}}$ vary considerably with energy and it is also not apparent that values of Y_j alone can ever be used to evaluate Y_{mixture} if $\sigma_j(E)$ and $(dx/dE)_j$ are not both known as a function of energy. One method of using the cross section data is to neglect the variation of $(dx/dE)_{\text{mixture}}$ with energy and take it outside the integral in equation (3). This is not as bad an approximation as at first sight appears because $\sigma_j(E)$ is zero below the threshold of endothermic reactions. An approximation of much wider applicability however, which doesn't neglect the variation of dE/dx with energy and which moreover enables the thick target data to be used alone, is as follows.

There is a wide range of atomic number and energy for which the variation of dE/dx with E is similar for different materials and one can write

$$(dE/dx)_j = K_{j1} (dE/dx)_1 \quad (4)$$

where K_{j1} is a constant, independent of energy, for the two materials j and 1 where 1 refers to some standard material.

Alternatively one can use the tabulated values

$$(dE/dpx)_j = \frac{\rho_1}{\rho_j} K_{j1} (dE/dpx)_1 \quad (5)$$

where ρ is the density.

From equation (2), using equation (4)

$$Y_j = \frac{n_j}{K_{j1}} \int_{E_a}^0 \sigma_j(E) (dx/dE)_1 dE \quad (6)$$

To get any further it is necessary to invoke the Bragg additivity law of stopping powers. Without this law or its equivalent nothing further can be done even with perfect knowledge of $\sigma_j(E)$, Y_j and $(dE/dx)_j$ for the constituents.

The Bragg additivity law can be written

$$\frac{1}{\rho_{\text{mixture}}} (dE/dx)_{\text{mixture}} = \frac{\sum_j (dE/dx)_j \cdot n'_j A_j / \rho_j}{\sum_j n'_j A_j} \quad (7)$$

Since

$$\rho_{\text{mixture}} = \frac{1}{N} \sum_j n'_j A_j$$

and

$$\rho_j = n_j A_j / N$$

where A_j is the atomic weight of element j and N is Avogadro's number, equation (7) can be simplified to

$$(dE/dx)_{\text{mixture}} = \sum_j (dE/dx)_j \cdot n'_j / n_j$$

or using equation (4)

$$(dE/dx)_{\text{mixture}} = (dE/dx)_1 \sum_j n'_j \cdot K_{j1} / n_j$$

Inserting this in equation (3)

$$Y_{\text{mixture}} = \frac{\sum_j n_j' \int_{E_0}^0 \sigma_j(E) \cdot (dx/dE)_1 dE}{\sum_j n_j' K_{j1}/n_j} \quad (8)$$

Referring to equation (6) the term under the integral is seen to be directly related to Y_j and so

$$Y_{\text{mixture}} = \sum_j \frac{n_j'}{n_j} K_{j1} Y_j / \sum_j \frac{n_j'}{n_j} K_{j1} \quad (9)$$

Thus we have expressed Y_{mixture} as a linear combination of Y_j 's the thick target yields of the constituents. The weighting factor for each Y_j is $n_j' K_{j1}/n_j$ and insofar as K_{j1} is a constant, independent of energy, for the materials j and l and the Bragg additivity law holds, the expression (9) is exact. No knowledge of $\sigma_j(E)$ is required if the thick target yields are known.

The extent to which K_{j1} can be taken as a constant will now be examined using published tables of stopping power for α -particles. There are three widely used compilations by Williamson, Boujot and Picard 1966, Northcliffe and Schilling 1970 and Ziegler 1977. The trends of stopping power are similar in all three compilations but they differ in detail. The tables of Williamson, Boujot and Picard have been used, in spite of being the oldest, because the data are presented more conveniently for our purposes.

We have evaluated

$$P = \frac{\frac{dE}{dx}_{j,E} / \frac{dE}{dx}_{\text{Oxygen},E}}{\frac{dE}{dx}_{j,5\text{MeV}} / \frac{dE}{dx}_{\text{Oxygen},5\text{MeV}}} \quad (10)$$

That is the stopping powers in different materials at different energies are compared with that of oxygen at 5 MeV. In Figure 1 are plotted contours on the Energy - Z plane showing the departure of P from unity. The surface $P=1$ corresponds to K_{j1} value which is constant and independent of energy where the subscript l in this case refers to oxygen. The contours define a saddle shaped surface with the greatest departures from unity occurring at low energies. It is apparent that there is a wide area over which approximate constancy of K_{j1} obtains. For instance if we take the $\pm 10\%$ contours the range of constancy of K_{j1} within these limits extends for Li from 1.9 MeV to well beyond 10 MeV whereas for Uranium the range is much less, 7.1 MeV to 3.4 MeV. The figures, it should be remembered, refer to the range of variation for the particular elements with respect to oxygen.

Figure 1 may also be used to decide on a best energy at which to evaluate K_{j1} for a particular case.

Suppose for instance we have 6.2 MeV α -particles incident on Beryllium oxide. The $Z=4$ abscissa in Fig. 1 intersects the -1.0% contour at 6.2 MeV, the $+2.5\%$ contour at 3.3 MeV, the $+5\%$ contour at 2.5 MeV and the $+10\%$ contour at 1.6 MeV. For the best average K between 6.2 MeV and 3.3 MeV the energy at which $Z=4$ intersects the contour $(-1+2.5)/2 = 0.75\%$ should be used (ie) at 4.4 MeV. Similarly between 6.2 and 2.5 MeV the point where $Z=4$ intersects the contour $(-1+5)/2 = 2\%$ should be used (ie) at 3.6 MeV. Finally for the best average between 6.2 and 1.6 MeV the point where $Z=4$ intersects the contour $(-1+10)/2 = 4.5\%$ should be used (ie) at 2.7 MeV. The insensitivity of the thick target yields to the low energy part of the integral in equation (6) makes it unnecessary to go to lower energies. The extreme limit of error involved in choosing the three energy ranges above is $(4.5-0.75)\%$ or $\pm 1.9\%$. When it is remembered that for Beryllium (dE/dpx) varies from 670 MeV/gm/cm² at 6.2 MeV to 1680 MeV/gm/cm² at 1.6 MeV it is apparent that the use of constant K_{j1} values as compared with assuming dE/dx is constant reduces the sensitivity of the computed yields to dE/dx by at least one order of magnitude.

Equation (9) is not expressed in the most convenient quantities for practical use*. It can be transformed as follows. If, instead of K_{j1} we use the quantity C_{j1} where

$$C_{j1} = \frac{\rho_1}{\rho_j} K_{j1} \quad (10)$$

and where ρ_1 and ρ_j are the densities of materials 1 and j. From equation (5)

$$(dE/dpx)_j = C_{j1} (dE/dpx)_1 \quad (11)$$

Using C_{j1} instead of K_{j1} in equation (9) we get

$$Y_{\text{mixture}} = \frac{\sum_j \frac{n_j}{n_j} \frac{\rho_j}{\rho_1} C_{j1} Y_j}{\sum_j \frac{n_j}{n_j} \frac{\rho_j}{\rho_1} C_{j1}} \quad (12)$$

If, in the case of a compound, the molecule contains a_j atoms of atomic weight A_j and there are n_m molecules/cc, then

$$n_j = n_m a_j$$

Substituting this in (12) and remembering that $\rho_j = n_j A_j / N$ and cancelling ρ_1 , n_m and N in the numerator and denominator.

$$Y_{\text{mixture}} = \frac{\sum_j a_j A_j C_{j1} Y_j}{\sum_j a_j A_j C_{j1}} \quad (13)$$

*I am indebted to Dr. B.W. Hooton for this comment.

The weighting factor for Y_j is now $a_j A_j C_{j1}$ where $a_j A_j$ is just proportional to the fraction by weight of element j in the compound or mixture. C_{j1} is obtained directly from stopping power tables using equation (11).

In conclusion it is worth mentioning two special cases of relation (13) which are often encountered.

- (1) A small admixture of a foreign material, 2, is present in an otherwise single component material.

From (13)

$$Y_{\text{mixture}} = \frac{a_1 A_1 Y_1 + a_2 A_2 C_{2,1} Y_2}{a_1 A_1 + a_2 A_2 C_{2,1}}$$

If therefore $a_2 A_2 C_{2,1} / a_1 A_1 \ll 1$

$$Y_{\text{mixture}} = Y_1 + \frac{a_2 A_2}{a_1 A_1} C_{2,1} Y_2 \quad (14)$$

This relation can be deduced directly from equation (3) without invoking the Bragg additivity law so is independent of it.

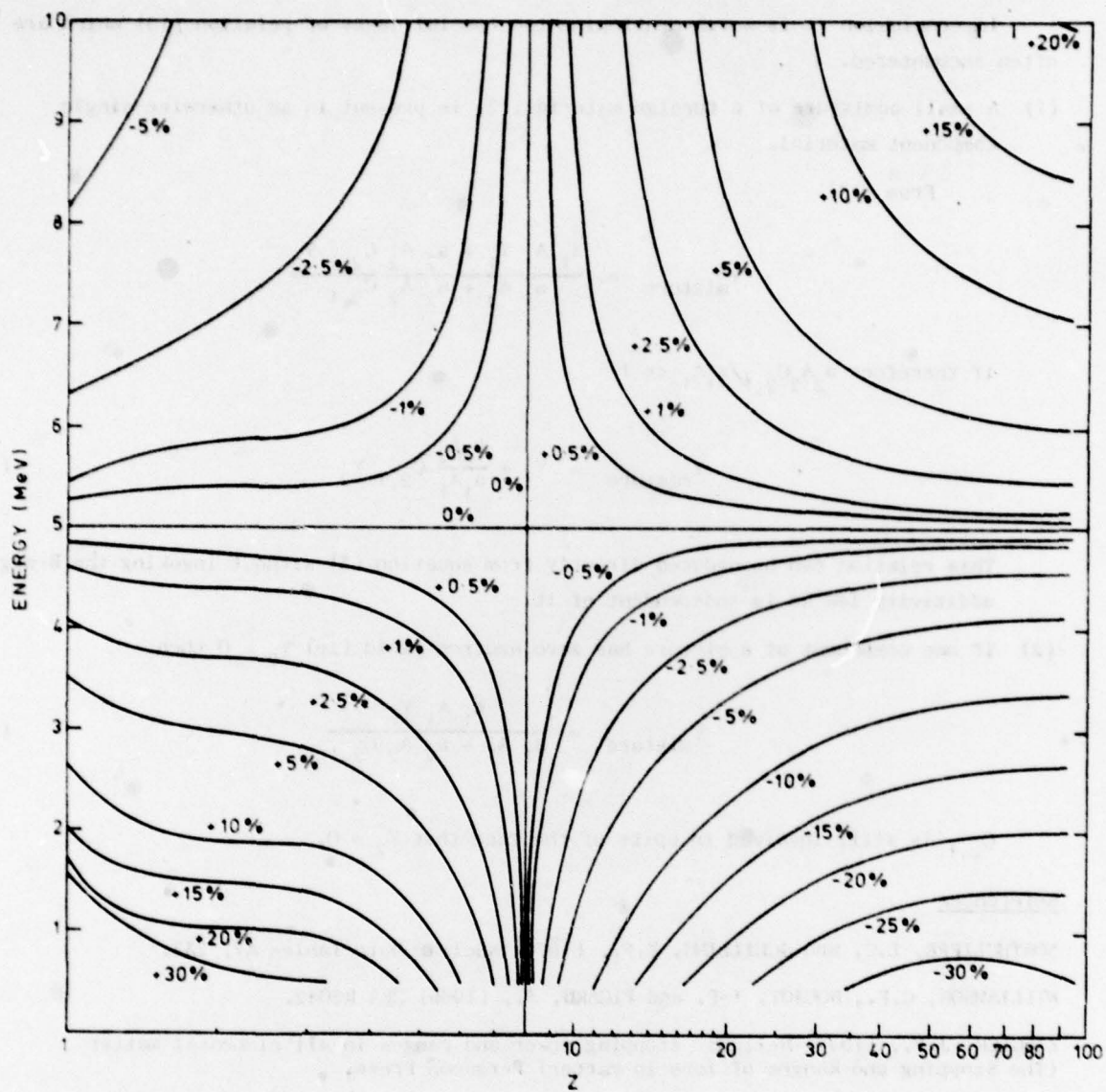
- (2) If one component of a mixture has zero neutron yield (ie) $Y_2 = 0$ then

$$Y_{\text{mixture}} = \frac{a_1 A_1 Y_1}{a_1 A_1 + a_2 A_2 C_{2,1}} \quad (15)$$

$C_{2,1}$ is still involved in spite of the fact that $Y_2 = 0$.

References

- NORTHCLIFFE, L.C. and SCHILLING, R.F., (1970) Nuclear Data Tables A7, 233.
 WILLIAMSON, C.F., BOUJOT, J-P. and PICARD, J., (1966) CEA R3042.
 ZIEGLER, J.F., (1977) Helium: stopping power and ranges in all elemental matter (The Stopping and Ranges of ions in matter) Pergamon Press.



AERE. R.9372 FIG. 1.